OBITUARY NOTICES

OF

FELLOWS DECEASED.

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SIR W. DE W. ABNEY, K.C.B., 1843—1920.

By the death, on December 2nd, 1920, of Sir William Abney there passed away one of the notable figures of the scientific world of the past forty years and one who will be long remembered as the great pioneer in the science of photography.

That it was worthy to be ranked as a pure science he always insisted, and on more than one occasion he deprecated the attitude of scientific men who used photography as a mechanical aid in their laboratories without endeavouring to understand it, and lamented that of 25,000 people who took photographs not more than one cared for, or knew anything about, the why and wherefore. However this may be—and we must be careful not to push the contention too far, or we shall find ourselves committed to the proposition that a knowledge of the chemistry of ink is a necessary part of the equipment of a writer—there is no doubt that in Abney's hands photography was an exact science, an offshoot from both chemistry and physics, concerned with the action of light upon all manner of bodies. With this science and its growth over many most fruitful years Abney's name must always be inseparably connected.

He was the eldest son of Canon E. H. Abney and was born on July 24, 1843. He was educated at Rossall School and entered the Royal Engineers through the Royal Military Academy in 1861. The first few years of his service, spent partly in India, were uneventful. In 1870 he returned to England and was stationed at Chatham, and in the next year was appointed Assistant to the Instructor in Telegraphy at the School of Military Engineering. Here he found himself in charge of a small photographic establishment and chemical laboratory, then constituting part of the Electrical School, and at once began the active prosecution of photographic researches. He formed classes of officers and men for studying the subject, and the first edition of his book, 'Instruction in Photography,' destined afterwards to reach its eleventh edition and to be the guide of innumerable students of the art, was printed at Chatham in 1871 as a small pamphlet for the use of his pupils.

In the previous year he had joined the Photographic Society and his first paper, on "The Application of Albumen to Photography," appeared in 1870. Under his energetic direction the photographic establishment soon outgrew the tutelage of the Instructor in Telegraphy and in 1874 a separate Chemical and Photographic School was formed of which he was given sole charge. From this time onward he was so continuously engaged upon the evolution of photography into an exact science that it would be hard to find any side of this field of knowledge which he had not made a subject of experiment and upon which he had not written fully and critically.

In 1874 the only extensively used photographic process was the collodion "wet plate." The gelatine dry plate, though actually first made in 1871, was

then very imperfect and the various dried collodion plates and collodion emulsions, in which Abney was one of the foremost experimenters, were only used by a few eager inquirers. In 1878–79, due mainly to advances made by Bennett and Abney in this country and van Monkhoven on the Continent, a rapid gelatine emulsion, in all main details identical with those used to-day, was first produced and the modern "instantaneous" photography made possible.

Abney rapidly established himself in a leading position as a practical exponent of the art and already in 1874 he was given complete charge of the arrangements for photographic observations of the Transit of Venus. He himself went to Egypt to observe it and brought back a great collection of views of temples and tombs, forming the basis of his book, 'Thebes and its Five Great Temples,' published in 1876.

Perhaps we may fairly claim that the first important memoir in which Abney showed his true quality and proved that he was indeed one of the elect to whom the why and wherefore are of the most basic importance, one who would never rest contented until he fully understood any process that went on under his observation, and saw that it is only by full understanding that further advance is made possible, was his paper "On the Alkaline Development of the Photographic Image" ('Phil. Mag.,' 1877). It had long been known—by whom the discovery was first made is still obscure—that where no free nitrate of silver was present, a considerably increased intensity of image was obtained with an alkaline developer, and it was generally assumed, though it cannot be claimed that there was much curiosity about it among the users, that the function of the alkali was to reduce the bromide, chloride, or iodide of silver, which had been acted upon by the light, to the metallic state. Abney showed, by a well planned and quite conclusive set of experiments, that the complete explanation was found in the fact that bromide of silver could not exist in close contiguity to the freshly reduced metal, but that sub-bromide was immediately formed, and similarly with chloride or iodide. This in its turn was again reduced to the metallic state by the developer, so that the image was eventually built up of metallic silver derived partly from those molecules of the salt acted upon by light, thereby reduced to the sub-salt, and further reduced to metal by the developer, and partly from the closely contiguous molecules, which were reduced to the sub-salt, not by light, but by contact with the freshly formed metal, and were then in their turn similarly acted on by the developer. The process was thus shown to be essentially different from the development of a plate with free nitrate of silver present, such as the ordinary wet-plate, where the salt reduced by light acts as a nucleus upon which metallic silver precipitated from the silver nitrate solution by the developer aggregates. As in the latter case the silver which forms the image is deposited out of a solution flowing freely over the plate there is almost no limit to the extent to which the image can be built up; the development and intensification can be pushed to the point where the high

lights are completely opaque. Whereas, with an emulsion or dry-plate, the position of each molecule of silver salt is fixed, and it remains in the same place after reduction. If, therefore, the development is pushed too far, the image will spread out, losing form, and, should the action be allowed to continue to an extreme, the whole plate will ultimately become covered with an opaque mass of reduced silver.

In this power of building up the image without allowing it to spread lies the outstanding advantage of the wet-plate over the emulsion process and makes it, even now, when the manufacture of dry plates has reached such a high degree of technical excellence, preferable for some purposes.

In 1877 Abney left Chatham and joined the Science and Art Department at South Kensington, then under the able direction of the late Sir John Donnelly, R.E. In that department he served twenty-six years rising by successive steps to the post of Principal Assistant Secretary, Board of Education, which he held from 1899 to 1903, when he retired upon the reorganisation of the department under the late Sir R. Morant. He afterwards, up to the time of his death, held the honorary post of Scientific Adviser to the Board.

On taking up his duty at South Kensington he at once established a laboratory, placed in one of those hideous iron buildings known to two generations of Londoners as the "Brompton Boilers," and for nearly thirty years that laboratory was the source and fount of an unending stream of original experiment and research.

It would be impossible to follow Abney's work in any detail. His papers, as recorded in the 'Royal Society Catalogue,' number over one hundred, and to these must be added a very large aggregate of minor, but still important papers, over seventy in the 'Photographic Journal' alone, and many in the 'Journal of the Camera Club' and in other cognate publications. Much of the material of these memoirs and addresses was embodied by him in book form and his three standard volumes, 'Instructions in Photography,' 'Photography with Emulsions,' and 'A Treatise on Photography,' went through many editions.

One of his most noteworthy achievements was the photography of the infra-red region of the spectrum. He began experimenting upon this in 1875 while still at Chatham, and eventually succeeded in obtaining an emulsion of bromide of silver in collodion, in which the salt was in such a condition of molecular aggregation, indicated by its giving a blue colour to transmitted light, that it was sensitive to rays beyond the visible portion of the spectrum down to about λ 12,000. With this he mapped the solar spectrum from A to λ 10,650 ("Bakerian Lecture," 'Phil. Trans.,' 1880). In a subsequent paper ('Phil. Trans.,' 1881), written in collaboration with the late Maj.-Gen. Festing, R.E., he extended the use of these hypersensitive plates to the study of the absorption spectra of organic bodies in the infra-red, leading to important indications of their molecular groupings. For these researches he was awarded the Rumford Medal in 1882.

In this, as indeed in all his work, Abney showed a high level of

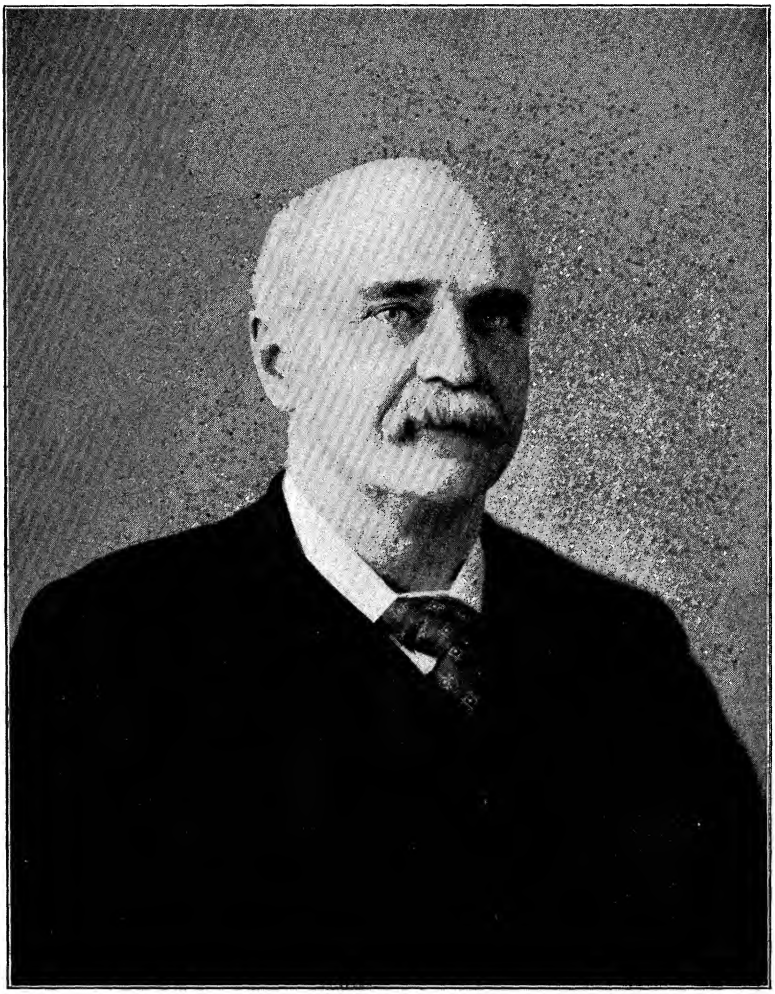
manipulative skill. Many subsequent experimenters have tried to prepare an infra-red sensitive emulsion upon his formula and very few have succeeded. That it can be done there is however no doubt whatever; much depends upon the quality of the collodion.

His researches upon the action of the spectrum on the silver salts naturally led to the study of colour photography and in later years he did an enormous amount of exploration in this branch of the subject. In his earlier years he was very sceptical as to the possibility of any practical three-colour process; thus in 1878 he was of opinion that it was "utterly impossible to secure monochromatic colours which are pure enough to give the truths of nature," and that efforts (to obtain true colours by combining negatives taken through three different colour screens) "are not to be followed with too much zeal by scientific photographers." Later on, when it had been shown by Ives that exact colour representation could be obtained, provided the screens for taking the negatives and for viewing or projecting the positives were properly selected, he set out in a masterly way the principles involved in three-colour work. ("Theory of Colour Vision applied to Modern Colour Photography," 'Proc. Royal Institution,' 1899).

In the Bakerian Lecture of 1886 on Colour Photometry he gave an account of some experiments, made in collaboration with Gen. Festing, on the measurement of the relative illuminating intensities of the different parts of the spectrum. This subject had occupied him for some time previously and both then and afterwards, almost up to the time of his death, he devoted himself with immense enthusiasm to the problems of colour vision and colour blindness. He repeated, by a different method and in a far more exhaustive and complete form, the delineation of the intensity curves done by Clerk-Maxwell in 1855–6 and by König in 1883–1901. Without entering upon the vexed question of the true mechanism of colour vision it will suffice to state here that Abney's observations appear to confirm in every detail the intensity curves for the three fundamental colour sensations derived by König and thus afford the most powerful support to the validity of the Young-Helmholtz trichromatic theory of colour vision and the corresponding explanation of the different varieties of colour blindness. In addition to some thirty papers and memoirs he summarised his investigations and reviewed the whole subject in a book, 'Researches in Colour Vision and the Trichromatic Theory,' 1913.

Another notable research carried out partly in England and partly at the Riffel and other stations in the High Alps was embodied in his memoirs on "The Transmission of Sunlight through the Earth's Atmosphere" ('Phil. Trans.,' 1887 and 1893).

In 1882 he had planned to go to Egypt to observe the Total Solar Eclipse of May 17 but was prevented by temporary ill-health. This was a great disappointment to him. He was a keen traveller and invariably spent his summer in the Swiss or Italian Alps where he pursued his photographic work both from the scientific and the artistic side, and was indefatigable in water-colour sketching.



W. de W. Abney

Abney was gifted in an unusual measure with true scientific insight, the power of seizing essentials, and we may almost say the power of predicting the future course of development. The point just mentioned of the three-colour process is probably the only one in which he was definitely wrong and where he failed to see the essentials of a problem. An extraordinary record for one whose output was so enormous! As an instance of his foresight it may be noted that in 1877, in a paper upon electric search-lights for military service, he advocated the employment of what is now known as a "flame" arc, using carbons charged with a calcium salt in order to give greater penetrative power to the beam by virtue of its ruddy colour.

All his experimental work was distinguished by its completeness and by the ingenuity and beauty of the means he devised. His early successes at Chatham in the improvement of photo-lithography; the invention of the "Abney ink"; the contrivance of the "Abney Level"; his methods for the photometry of both monochromatic and coloured lights; his precise measurements of the opacity of the deposit on the photographic plate leading to a statement of the law connecting density and exposure; his expedients for estimating the coronal light during a solar eclipse, for determining the transparency of the atmosphere and for the comparative evaluation of sunlight, starlight and skylight, were all marked with these same general features. He never used apparatus more elaborate than the actual conditions demanded; much of his work was indeed done with appliances which might to the ordinary spectator seem perplexingly crude, but when fully understood it was always clear that the conditions for the degree of accuracy aimed at were amply fulfilled. He had, in truth, no real liking for researches necessitating any extreme or meticulous refinement, he was too well aware what an expenditure of time these involve and was conscious of many more pressing things to be done. When a suggestion was made to amplify any line of his work with added detail, to carry the results to a higher degree of precision, he not infrequently advised "leaving it to the Germans." He was quite content with putting the problem upon the right lines and pursuing the conclusions up to the productive point; others could follow later into the arid regions of the third decimal place if they so desired and had no better use for their energies.

He was elected a Fellow in 1876, and served at different times in the offices of President of the Royal Astronomical, the Royal Photographic and the Physical Societies; also as Chairman of the Royal Society of Arts. He was President of Section A (Physics) at the British Asociation in 1889 where he gave an address summarising all that was known of the theory of photographic action. He was created K.C.B. in 1900. He married twice; first in 1864, Agnes Matilda, daughter of E. W. Smith, of Tickton Hall, Yorks, who died in 1888; secondly, in 1890, Mary Louisa, daughter of the Rev. G. N. Meade, of Scarborough on Hudson, U.S.A., who survives him.

He leaves one son and two daughters of the first marriage, one being the wife of Rear-Admiral Sir Reginald Hall; and one daughter of the second.

E. H. G-H.

ROBERT BELLAMY CLIFTON, 1836—1921.

Robert Bellamy Clifton was born at Gedney, in Lincolnshire, on March 13, 1836. He was the only son of Robert Clifton, a landowner in the district. At an early age he went to school at Peterborough, and afterwards at Brighton; then, after a period at University College, London, he entered at St. John's College, Cambridge, and took his degree as Sixth Wrangler in 1859. Canon J. M. Wilson, afterwards headmaster of Clifton, was Senior. Prof. Jack, who succeeded Clifton at Manchester, Prof. Adams, of King's College, London, and Dr. Stone, afterwards Radcliffe Observer, were in the same Tripos. Clifton was second Smith's Prizeman, Wilson being first, and it is clear from the testimonials he received when a candidate, in 1865, for the Professorship of Experimental Philosophy at Oxford, that his place in the Tripos was not thought by his contemporaries to represent his real merits.

He was elected a Fellow of his College shortly after taking his degree, and in 1860 became the first Professor of Natural Philosophy in Owen's College, Manchester. Roscoe was then Professor of Chemistry and in his 'Life and Experiences' writes:—"After a time R. B. Clifton, then a distinguished young Cambridge man, was appointed Professor of Physics, and this was the first step towards the expansion of the College in a scientific direction. Clifton soon became most popular; his lectures were admirable and enabled me to dispense with teaching any portion of his subject."

Clifton threw himself heartily into his work of teaching, giving annually two courses of experimental lectures, one course on elementary applied mathematics, with a short experimental course on the same subject, and occasional lectures involving higher mathematics. In his application for the Oxford chair, he states his conviction of the possibility of rendering intelligible to students possessing only the most elementary knowledge of mathematics, "by experimental lectures only, the results, both severally and in their mutual relations, of a thorough study of the science, provided the order in which the results are presented accords with that indicated by a strict mathematical investigation of the subject." This conviction he retained through life: it guided his work at Oxford, and the methods of instruction followed in the Clarendon Laboratory.

In 1865 he was appointed to the Professorship of Experimental Philosophy in the University of Oxford. The testimonials with which his application was supported form a striking collection. Stokes, Thomson, Adams, Joule, Roscoe, Bunsen, Kirchhoff, and Whewell write in the highest terms of his work and of the expectations formed of his ability and the hopes that in the freedom of Oxford it would find greater scope for research than had been possible at Manchester.

His first work in his new position was to design and build the Clarendon Laboratory, the first built in Europe for the special purpose of experimental instruction in Physics.

The money for its erection came from the proceeds of the sale of Clarendon's 'History of the Great Rebellion,' and in a letter Dr. Madan, formerly Bodley's Librarian, writes:

"Henry, Lord Cornbury, grandson of the historian, the first Earl of Clarendon, left many of the first earl's papers to trustees, with the direction that the money from the sale or publication of his papers should be the nucleus of a fund for an academy for riding or other exercises at Oxford. This was in 1751. But he died before his father. However, a sister carried out his intention, but the money was left to accumulate. In 1860 his trustees found they had £10,000, but as the University didn't need a riding school, but did badly want a laboratory for physical science, his trustees, by that wisdom which belongs only to lawyers and trustees, promptly erected the Clarendon Laboratory, which Prof. Clifton was the first to administer in 1872."

To design the Laboratory was his first task. The architect was responsible for the exterior; the fittings, down to minute details, were carried out from his own working drawings, and then, the building being complete, came its equipment with apparatus. Much of this was designed and redesigned by him until perfection, or something approaching it, was reached, and so much loving care had been spent on an instrument that it needed to be kept jealously under lock and key, taken out from time to time to be dusted and cleaned, possibly to be used in lecture, but entrusted never to the careless handling of a student of Physics.

At first he took some share in the practical instruction in the Laboratory, but for some time past this was left to his demonstrators. He was at the Laboratory every day, and generally went round and talked to the men, occasionally giving a demonstration on some special instrument, such as the Michelson interferometer.

Mechanics were outside his scope; until there was a special department for Electricity his lectures included a course on Electricity and Magnetism. After that they were chiefly concerned with Acoustics or Optics. His great idea was that his lectures should give his class instruction in giving experimental lectures; in the summer term he showed his men the solar spectrum in great detail with a grating spectroscope. He also lectured on the optical properties of crystals and the phenomena of polarisation, doing everything with minute care and great deliberation.

Of research work there was but little: the Laboratory was intended for teaching. This was the more unfortunate as the future careers of his pupils—among them may be mentioned Sir Arthur Rücker, Prof. Reinold, Sir Lazarus Fletcher, and others—have shown how competent they were to carry on research and advance knowledge, not merely by their teaching, but by original investigations.

Almost the only great piece of research appearing from the Clarendon Laboratory in his time was Boys' determination of the constant of gravitation, an investigation in which the author received the most cordial help and the most valuable support from the Professor.

To return to his own work. From 1879 to 1886 he was a member, along with Prof. Tyndall, Sir Frederick Abel, and others of the Royal Commission on Accidents in Mines, and assisted in carrying out a number of the experiments on which the conclusions of the Commission were based. He designed a safety lamp, which is described in Appendix XXIV of the Report and formed the subject of a number of the investigations. The general idea was to use the products of combustion so as to shield the flame from contact with an inflammable mixture.

He was elected a Fellow of the Royal Society in 1868, served three times on the Council, and was Vice-President in 1896–98. In 1869 he was elected a Fellow of Merton College, and at the time of his death he was an Honorary Fellow of Wadham. He published but little, partly because of his absorption in his teaching, but mainly, perhaps, as a consequence of his own high ideal of the level which published work should reach.

The 'Proceedings of the Literary and Philosophical Society of Manchester' for 1860–62 contain a paper, written jointly with Sir Henry Roscoe, on the "Effect of Increased Temperature upon the Nature of the Light emitted by the Vapours of certain Metals or Metallic Compounds," in which the suggestion, in the main correct, is made, that the flame spectra are due to the oxides of the metals, and not to the elements themselves. Another paper of interest, also published at Manchester, is one entitled "An Attempt to refer some Phenomena attending the Emission of Light to Mechanical Principles." It contains an early attempt to apply to the production of a spectrum some of the elementary facts of the kinetic theory of gases. His paper on "The Difference of Potential produced by the Contact of different Substances" ('Roy. Soc. Proc.,' vol. 26, 1877), was a useful contribution to the discussion then in progress as to the seat of the electromotive force in an electric circuit.

From early days he collected mathematical books of historical interest, building in later years a library, which he called the "Folly," on to his home at Oxford, to house these. He had a large collection of old Euclids, arithmetics, etc., and knew more about their contents than bibliophiles usually do. There is a story that, while still an undergraduate, Whewell sent for him and asked him his price for a copy of Calendri's 'De Arithmetica Opusculum' (Florence, 1491), which he had bought for 2*s.* 6*d.* Clifton declined to part with it, and Whewell, who wanted the book for a friend, said, "Well, I have done my duty by my friend, but now—you keep that book!"

Throughout his life he was a most kind friend to colleagues and pupils alike—a courteous gentleman, hospitable, and ready to help. A Manchester paper, referring to his successor in the Chair of Natural Philosophy, writes: "There is in him little of that easy grace which marked Mr. Clifton, making him most interesting and dignified when lecturing from a seat on the table, or when, having thrown away his gown, he worked with a vigour quite astounding to the audience at some laborious experiment"; the grace and dignity he retained to the end. According to "Who's Who," his recreation

was "Work." The work included the supervision of a considerable property in Lincolnshire, which he had inherited from his father, and the welfare of his tenants, specially during a period of agricultural depression, which followed his father's death in 1873, was always uppermost in his thoughts. During term time, until he resigned his Professorship, he kept up a curious habit of dividing his day and night; he would rest in his chair after dinner for a time, and then when the household had gone to bed, he would start his work and go right on until 7.30 or 8 A.M. Then he went to bed for a couple of hours or so, and by 11 he was usually to be found at the Laboratory.

Prof. Clifton married, in 1862, Miss Butler, of Brighton, and leaves three sons and a daughter. Until 1893 their house at Oxford was always open to their friends, and they made a point of entertaining the men working at the Clarendon every term. In that year Mrs. Clifton had a serious illness, and for a time was cut off from all hospitality; before her death, in 1917, however, she recovered to a considerable extent, and the Sunday afternoon receptions again became quite an institution.

Oxford has changed; many of those who owed their training and their fortunes to Prof. Clifton have passed away, but the Clarendon Laboratory remains as a memorial to one who, though adopted, became her true and loyal son.

R. T. G.

SIR LAZARUS FLETCHER, 1854—1921.

Sir Lazarus Fletcher was born at Salford, March 3, 1854. He was the eldest of a family of six sons and two daughters, one of whom, the Rev. Mark Fletcher, F.G.S., is lecturer on Mineralogy at the Armstrong College, University of Durham, Newcastle-upon-Tyne.

He did not come from a scientific family, nor was it well-to-do; hence his attainments were not due to favourable circumstances in early life, but to his own love of work and learning, and the disadvantages which he had experienced made him very sympathetic with struggling students and ever ready to help them to the utmost of his power.

He was educated at the Manchester Grammar School under Dr. Marshall Watts, and for a time Mr. Francis Jones for Chemistry, Mr. Angell for Physics, and the Rev. J. Chambers for Mathematics, from whom he may have received his scientific bent as well as instruction. He changed over from the Science to the Mathematical VIth in 1871, and was equally good in both divisions, and while still at school he gained, in 1872, the Gold Medal for Mechanics and a Bronze Medal for Mathematics, and several

first class certificates in other subjects at the National Examinations held by the Science and Art Department at South Kensington, also an open Science Scholarship (the Brackenbury) at Balliol, in 1872. He joined his College the same year and took a "highly distinguished" first class in Mathematical Mods. in 1874, and first classes in the Final Examinations in both the Natural Science and Mathematical Schools in 1876.

He was appointed Demonstrator in the Clarendon Physical Laboratory (1875–77) under Prof. R. B. Clifton, F.R.S., and was elected to the Millard Lectureship in Physics at Trinity College, Oxford, in 1877–78; also in 1877 to a Fellowship at University College, Oxford, which he had to resign in 1880 on account of his marriage, but was made an Hon. Fellow in 1910.

On account of his knowledge of Crystallography and Mineralogy he was appointed Assistant in the Mineralogy Department at the British Museum in 1878, under Prof. Nevil Story Maskelyne, F.R.S., in succession to Prof. W. G. Lewis, who had resigned, and he was promoted to the Keepership of the Department on Prof. Maskelyne's resignation in 1880, and held it till 1909.

Almost immediately after his appointment he had to carry out the removal of the great collections of Minerals and Meteorites from Bloomsbury and re-arrange them in the new buildings at South Kensington: probably the largest, finest and most valuable collection in the world. This was a most arduous and anxious task, not only on account of the vast number of specimens which had to be packed up for the journey and unpacked again and re-arranged, but also on account of the fragility of many, the very heavy weight of some, the great intrinsic value of the gold, diamonds and gems, and the priceless scientific value of the many rare and unique specimens, all of which, of course, had to be guarded against loss and theft, especially while out of their cases. He had to design new show-cases and alter others to adapt them to their new positions. Next he prepared Guides and Handbooks to these collections, which are models for simplicity and accuracy, and are helpful not only to the ordinary visitor but also to the student.

He was Examiner in Natural Science at the Oxford Public Examinations, 1880, and for the Cambridge Natural Science Tripos in 1882–3, 1889–91, 1896–7. He was elected a Fellow of the Royal Society in 1889, and served on the Council 1895, 1897, and 1910–12, and as a Vice-President in 1910–12, also a Vice-President of the Geological Society 1890–92; of the Physical Society 1895–7; President of the Mineralogical Society of Great Britain and Ireland 1885–8, and General Secretary 1888–1909; and President of the Geological Section, British Association, Oxford, 1894, and was awarded the Wollaston Medal of the Geological Society, 1912.

In presenting the Wollaston Medal of the Geological Society to Fletcher in 1912, the President, Prof. W. W. Watts, F.R.S., in the course of his speech, said "that the Council desired to place you in company with Bischof, Naumann, Dana, von Hauer, Descloizeaux, Story Maskelyne and von Groth.

. . . . I ask you to accept the Wollaston Medal, not for the sake of yourself and your work alone, but as a token of acknowledgement of the Science of Geology of part of her debt to the science which you so worthily represent in our country."

In the course of his acknowledgement, and in reference to the President's appreciative remarks upon his Optical Indicatrix paper, Fletcher said:—"After the work of Fresnel, Sir W. Hamilton, MacCullagh, Sylvester, and others, it was some time before he could convince himself that there was anything left for the gleaner, and that the relationship, if true, would have been discovered long ago." The relationship referred to is "that for every biaxial crystal to each point on the ellipsoid there corresponds a single ray of light, with three physical characters, viz., the direction and velocity of transmission and the plane of polarization—all definitely and simply related to the geometrical characters of the ellipsoid at that point."

He was appointed Director of the Natural History Branch of the British Museum in 1909, on the retirement of Sir Ray Lankester, K.C.B., was knighted in 1916, and retired in 1919.

It was during his directorship that the first guide-lecturer upon the Collections, Mr. T. H. Leonard, B.Sc., was appointed, and, in consequence, many visitors receive instruction which enables them to take an intelligent interest in what they see, instead of wandering aimlessly through the galleries and coming away with a museum headache. This innovation has been a most successful departure.

He was a Member of the Boards of Electors of Oxford and Cambridge to the Professorships of Mineralogy of both Universities. In addition to those already mentioned, he received numerous home and foreign honours, viz., he was made Corresponding Member of the Royal Society of Göttingen, the Academy of Sciences, Munich, and the New York Academy of Sciences. Hon. Member of the Soc. Científica, Antonio Alzate, Mexico; the Association Sci. et d'Enseignement Médical Complémentaire; the Selborne Society; the Hertfordshire Natural History Society and Field Club; the Ealing Science and Microscopical Society; and the Museums Association, and Hon. LL.D. of St. Andrews, and Hon. A.M. and Ph.D. of Berlin.

Over and above his administrative duties at the Natural History Museum, where he had larger collections to look after and a larger staff to supervise than when the Collections were at Bloomsbury, he found time for extra work in the way of research, and especially upon meteorites.

His services also to the Mineralogical Society were very great; he was President of it in 1885 to 1888, and its Hon. Secretary from 1888 to 1909; and, in recognition of the whole-hearted way he worked for it and ensured its success, the members and other friends subscribed for his portrait in oils and presented it to him in 1912.

The following are among other more important publications:—"An Introduction to the Study of Meteorites," 1881; "An Introduction to the Study of Minerals," 1884; "An Introduction to the Study of Rocks," 1895.

These were written as Guides to the Natural History Museum Collections, and ran through several editions. "On the Dilatation of Crystals," 'Phil. Mag.,' 1880; and that on "The Optical Indicatrix and Transmission of Light," 'Min. Mag.,' 1892, already referred to, in which he pointed out an error of Fresnel that had been copied by writers of text-books, etc., but is now corrected, thanks to Fletcher's investigations.

He delivered a notable Presidential Address to the Geological Section of the British Association—"On the Progress of Mineralogy and Crystallography," 1894, and also wrote the article on Meteorites in the 'Encyclopædia Britannica,' and prepared the 'Instructions for Collecting Rocks and Minerals in the Antarctic Manual,' 1901.

Most of his papers were read before the Mineralogical Society, and published in the 'Mineralogical Magazine,' and were upon Meteorites. He also gave a course of lectures on Meteorites in 1895, and again in 1904, at the Royal Institution.

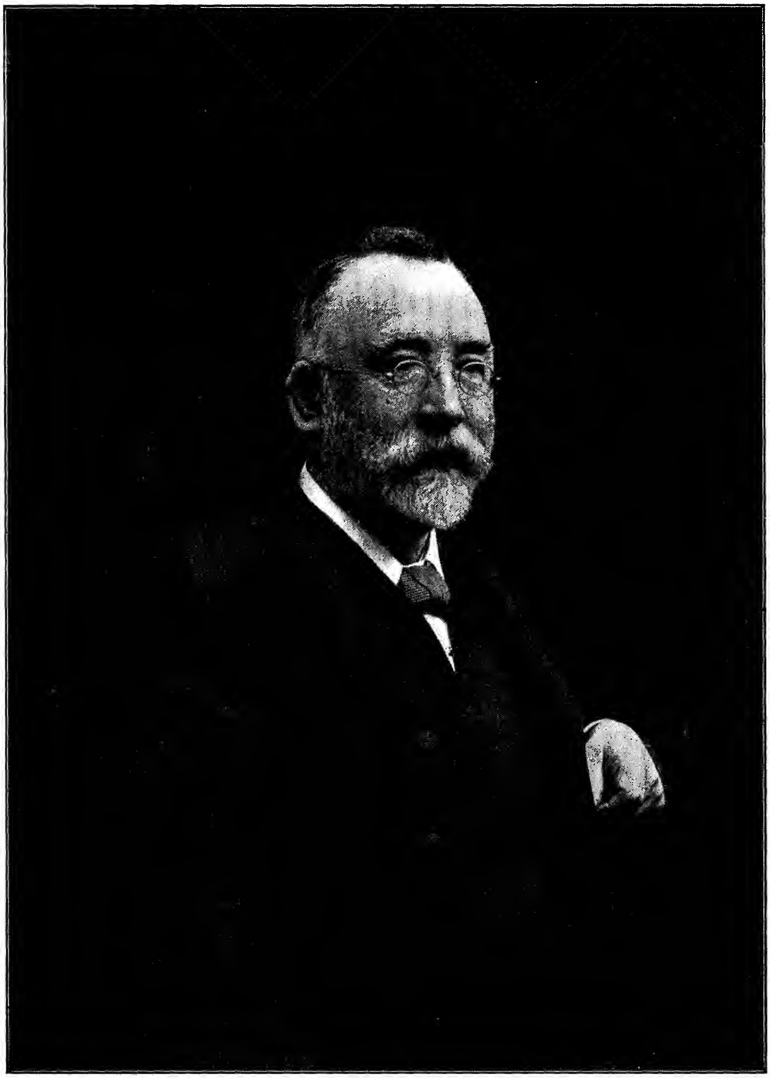
Fletcher ever displayed great simplicity and charm of manner, and was of a most kindly and sympathetic disposition; the writer never heard him make a disparaging remark, or say an unkind word, about anyone; he always took a lenient view and gave credit for the best intentions; he was true to his friends, and would take any amount of trouble on their behalf; hence, for these and other reasons, he was held by them in affectionate regard.

He was gifted with humour, and would often sum up a matter by a whimsical humorous remark with a gentle smile and a merry twinkle, the piquancy of the sally often heightened by the assumption of a slight and pleasant Lancashire intonation. He was a model correspondent, even when far from well, and when unable to write himself he dictated, shortly before his death, quite a long letter to the present writer, with his good wishes for the New Year. He had a serious illness in 1906, and never really recovered his full strength and energy.

On his retirement from the Directorship he had to his own great regret and that of his friends, to resign his membership of the Athenæum and Savile Clubs, as his lessened income, the crushing income tax, and high cost of living in London, necessitated reduced expenditure; he therefore went to live at Ravenstonedale, a little village in Westmoreland. He was thus deprived of many social and other advantages, especially of the society of his friends, and of participation in the work of scientific societies and institutions.

He died at Grange-over-Sands on January 6 last, just before the date he had fixed for returning home, and was buried at Ravenstonedale on the 12th. On account of the distance but few of his many friends, to their regret, were able to attend his funeral. Lady Fletcher and his daughter survive him.

A. L.



L. Fletcher

SRINIVASA RAMANUJAN, 1887—1920.

I.

SRINIVASA RAMANUJAN, who died at Kumbakonam on April 26, 1920, was elected a Fellow of the Society in 1918. He was not a man who talked much about himself, and until recently I knew very little of his early life. Two notices, by P. V. Seshu Aiyar and R. Ramachandra Rao, two of the most devoted of Ramanujan's Indian friends, have been published recently in the 'Journal of the Indian Mathematical Society'; and Sir Francis Spring has very kindly placed at my disposal an article which appeared in the 'Madras Times' of April 5, 1919. From these sources of information I can now supply a good many details with which I was previously unacquainted. Ramanujan (Srinivasa Iyengar Ramanuja Iyengar, to give him for once his proper name) was born on December 22, 1887, at Erode in southern India. His father was an accountant (*gumasta*) to a cloth merchant at Kumbakonam, while his maternal grandfather had served as *amin* in the Munsiff's (or local judge's) Court at Erode. He first went to school at five, and was transferred before he was seven to the Town High School at Kumbakonam, where he held a "free scholarship," and where his extraordinary powers appear to have been recognised immediately. "He used", so writes an old schoolfellow to Mr. Seshu Aiyar, "to borrow Carr's 'Synopsis of Pure Mathematics' from the College library, and delight in verifying some of the formulæ given there. . . . He used to entertain his friends with his theorems and formulæ, even in those early days. . . . He had an extraordinary memory, and could easily repeat the complete lists of Sanscrit roots (*atmanepada* and *parasmepada*); he could give the values of $\sqrt{2}$, π, e, ... to any number of decimal places. . . . In manners, he was simplicity itself. . . ."

He passed his matriculation examination to the Government College at Kumbakonam in 1904, and secured the "Junior Subraniam Scholarship". Owing to weakness in English, he failed in his next examination and lost his scholarship, and left Kumbakonam, first for Vizagapatam and then for Madras. Here he presented himself for the "First Examination in Arts" in December, 1906, but failed and never tried again. For the next few years he continued his independent work in mathematics, "jotting down his results in two good-sized note-books"; I have one of these note-books in my possession still. In 1909 he married, and it became necessary for him to find some permanent employment. I quote Mr. Seshu Aiyar:—

To this end, he went to Tirukoilur, a small sub-division town in South Arcot District, to see Mr. V. Ramaswami Aiyar, the founder of the Indian Mathematical Society, but Mr. Aiyar, seeing his wonderful gifts, persuaded him to go to Madras. It was then after some four years' interval that Mr. Ramanujan met me at Madras, with his two

well-sized note-books referred to above. I sent Ramanujan with a note of recommendation to that true lover of mathematics, Dewan Bahadur R. Ramachandra Rao who was then District Collector at Nellore, a small town some eighty miles north of Madras. Mr. Rao sent him back to me saying it was cruel to make an intellectual giant like Ramanujan rot at a mofussil station like Nellore, and recommended his stay at Madras, generously undertaking to pay Mr. Ramanujan's expenses for a time. This was in December, 1910. After a while, other attempts to obtain for him a scholarship having failed, and Ramanujan himself being unwilling to be a burden on anybody for any length of time, he decided to take up a small appointment under the Madras Port Trust in 1911.

But he never slackened his work at mathematics. His earliest contribution to the 'Journal of the Indian Mathematical Society' was in the form of questions communicated by me in vol. 3 (1911). His first long article on "Some Properties of Bernoulli's Numbers" was published in the December number of the same volume. Mr. Ramanujan's methods were so terse and novel, and his presentation was so lacking in clearness and precision, that the ordinary reader, unaccustomed to such intellectual gymnastics, could hardly follow him. This particular article was returned more than once by the editor before it took a form suitable for publication. It was during this period that he came to me one day with some theorems on Prime Numbers, and when I referred him to Hardy's Tract on 'Orders of Infinity,' he observed that Hardy had said on p. 36 of his Tract "the exact order of $\rho(x)$ [defined by the equation

$$\rho(x) = \pi(x) - \int_2^x \frac{dt}{\log t},$$

where $\pi(x)$ denotes the number of primes less than x] has not yet been determined," and that he himself had discovered a result which gave the order of $\rho(x)$. On this I suggested that he might communicate his result to Mr. Hardy, together with some more of his results.

This passage brings me to the beginning of my own acquaintance with Ramanujan. But before I say anything about the letters which I received from him, and which resulted ultimately in his journey to England, I must add a little more about his Indian career. Dr. G. T. Walker, F.R.S., Head of the Meteorological Department, and formerly Fellow and Mathematical Lecturer of Trinity College, Cambridge, visited Madras for some official purpose some time in 1912, and Sir Francis Spring, K.C.I.E., the Chairman of the Madras Port Authority, called his attention to Ramanujan's work. Dr. Walker was far too good a mathematician not to recognise its quality, little as it had in common with his own. He brought Ramanujan's case to the notice of the Government and the University of Madras. A research studentship, "Rs. 75 *per mensem* for a period of two years", was awarded him, and he became, and remained for the rest of his life, a professional mathematician.

II.

Ramanujan wrote to me first on January 16, 1913, and at fairly regular intervals until he sailed for England in 1914. I do not believe that his letters were entirely his own. His knowledge of English, at that stage of his life, could scarcely have been sufficient, and there is an occasional phrase which is hardly characteristic. Indeed, I seem to remember his telling me

that his friends had given him some assistance. However, it was the mathematics that mattered, and that was very emphatically his.

"Dear Sir, Madras, January 16, 1913.

"I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about twenty-three years of age. I have have had no university education, but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at mathematics. I have not trodden through the conventional regular course which is followed in a university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general, and the results I get are termed by the local mathematicians as 'startling'.

"Just as in elementary mathematics you give a meaning to a^n when n is negative and fractional to conform to the law which holds when n is a positive integer, similarly the whole of my investigations proceed on giving a meaning to Eulerian Second Integral for all values of n. My friends who have gone through the regular course of university education tell me that $\int_0^\infty x^{n-1}e^{-x}dx = \Gamma(n)$ is true only when n is positive. They say that this integral relation is not true when n is negative. Supposing this is true only for positive values of n, and also supposing the definition $n\Gamma(n) = \Gamma(n+1)$ to be universally true, I have given meanings to these integrals, and under the conditions I state the integral is true for all values of n negative and fractional. My whole investigations are based upon this, and I have been developing this to a remarkable extent, so much so that the local mathematicians are not able to understand me in my higher flights.

"Very recently I came across a tract published by you styled 'Orders of Infinity', in p. 36 of which I find a statement that no definite expression has been as yet found for the no of prime nos less than any given number. I have found an expression which very nearly approximates to the real result, the error being negligible. I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value, I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get, but I have indicated to the lines on which I proceed. Being inexperienced, I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

"I remain, Dear sir, Yours truly,

"S. RAMANUJAN.

"P.S.—My address is S. Ramanujan, Clerk Accounts Department, Port Trust, Madras, India."

I quote now from the "papers enclosed," and from later letters.

"The following are a few examples from my theorems :—

(1) The nos of the form $2^p 3^q$ less than $n = \frac{1}{2}\frac{\log(2n)\log(3n)}{\log 2 \log 3}$ where p and q may have any positive integral value including 0.

(2) Let us take all nos containing an odd no of dissimilar prime divisors, viz. :—

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 30, 31, 37, 41, 42, 43, 47, etc.

(*a*) The no of such nos less than $n = \frac{3n}{\pi^2}$.

$$(b)\ \frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots+\frac{1}{30^2}+\frac{1}{31^2}+\ldots=\frac{9}{2\pi^2}.$$

$$(c)\ \frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\text{ etc.}=\frac{15}{2\pi^4}.$$

(3) Let us take the no of divisors of natural nos, viz. :—

1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, etc. (1 having 1 divisor, 2 having 2, 3 having 2, 4 having 3, 5 having 2, etc.).

The sum of such nos to n terms

$$= n(2\gamma - 1 + \log n) + \tfrac{1}{2} \text{ of the no of divisors of } n,$$

where $\gamma = 0{\cdot}5772156649 \ldots$, the Eulerian Constant.

(4) 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, etc., are nos which are either themselves sqq. or which can be expressed as the sum of two sqq.

"The no of such nos greater than A and less than B

$$= K\int_A^B \frac{dx}{\sqrt{\log x}} + \theta(B) \quad \text{where} \quad K = 0{\cdot}764 \ldots$$

and $\theta(x)$ is very small when compared with the previous integral. K and $\theta(x)$ have been exactly found though complicated. . . ."

It may be well that I should interpolate here a few remarks concerning Ramanujan's researches in this particular field.

Ramanujan's theory of primes was vitiated by his ignorance of the theory of functions of a complex variable. It was (so to say) what the theory might be if the Zeta-function had no complex zeros. His methods of proof depended upon a wholesale use of divergent series. He disregarded entirely all the difficulties which are involved in the interchange of double limit operations; he did not distinguish, for example, between the sum of a series Σa_n and the value of the Abelian limit

$$\lim_{x \to 1} \Sigma a_n x^n,$$

or that of any other limit which might be used for similar purposes by a

modern analyst. There are regions of mathematics in which the precepts of modern rigour may be disregarded with comparative safety, but the Analytic Theory of Numbers is not one of them, and Ramanujan's Indian work on primes, and on all the allied problems of the theory, was definitely wrong. That his proofs should have been invalid was only to be expected. But the mistakes went deeper than that, and many of the actual results were false. He had obtained the dominant terms of the classical formulæ, although by invalid methods; but none of them are such close approximations as he supposed.

This may be said to have been Ramanujan's one great failure. And yet I am not sure that, in some ways, his failure was not more wonderful than any of his triumphs. Consider, for example, problem (4). The dominant term is

$$\frac{KB}{\sqrt{\log B}} \tag{α}$$

(I adhere to Ramanujan's notation): this result was first obtained by Landau in 1908. The error is of order $B(\log B)^{-\frac{3}{2}}$; and this is so far in agreement with Ramanujan's assertion. Ramanujan, however, implies much more,* and what he implies is definitely false: his integral does not represent the number of numbers in question more accurately than Landau's formula (α). However, Ramanujan had none of Landau's weapons at his command; he had never seen a French or German book; his knowledge even of English was insufficient to enable him to qualify for a degree. It is sufficiently marvellous that he should have even dreamt of problems such as these, problems which it has taken the finest mathematicians in Europe a hundred years to solve, and of which the solution is incomplete to the present day.

"... IV. Theorems on integrals. The following are a few examples:—

(1) $$\int_0^\infty \frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2} \ldots \text{etc. } dx$$

$$= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(a+\frac{1}{2})}{\Gamma(a)} \cdot \frac{\Gamma(b+1)}{\Gamma(b+\frac{1}{2})} \cdot \frac{\Gamma(b-a+\frac{1}{2})}{\Gamma(b-a+1)}.$$

......

(3) If $$\int_0^\infty \frac{\cos nx}{e^{2\pi\sqrt{x}}-1}\, dx = \phi(n),$$

then $$\int_0^\infty \frac{\sin nx}{e^{2\pi\sqrt{x}}-1}\, dx = \phi(n) - \frac{1}{2n} + \phi\left(\frac{\pi^2}{n}\right)\sqrt{\frac{2\pi^3}{n^3}}.$$

* See his statement concerning the order of $\theta(x)$, quoted below from his letter of February 27, 1913.

$\phi(n)$ is a complicated function. The following are certain special values :

$$\phi(0)=\frac{1}{12};\ \phi\left(\frac{\pi}{2}\right)=\frac{1}{4\pi};\ \phi(\pi)=\frac{2-\sqrt{2}}{8};\ \phi(2\pi)=\frac{1}{16};$$

$$\phi\left(\frac{2\pi}{5}\right)=\frac{8-8\sqrt{5}}{16};\ \phi\left(\frac{\pi}{5}\right)=\frac{6+\sqrt{5}}{4}-\frac{5\sqrt{10}}{8};\ \phi(\infty)=0;$$

$$\phi\left(\frac{2\pi}{3}\right)=\frac{1}{3}-\sqrt{3}\left(\frac{3}{16}-\frac{1}{8\pi}\right).$$

(4) $$\int_0^\infty \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)\ldots\text{etc.}}=\frac{\pi}{2(1+r+r^3+r^6+r^{10}+\text{etc.})},$$

where 1, 3, 6, 10, etc., are sums of natural nos.

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(6) $\displaystyle\int_0^\infty \frac{\tan^{-1}\dfrac{2nz}{n^2+x^2-z^2}}{e^{2\pi z}-1}\,dz$ can be exactly found if $2n$ is any integer and x any quantity."

......

"V. Theorems on summation of series;* *e.g.*,

(1) $$\frac{1}{1^3}\cdot\frac{1}{2}+\frac{1}{2^3}\cdot\frac{1}{2^2}+\frac{1}{3^3}\cdot\frac{1}{2^3}+\frac{1}{4^3}\cdot\frac{1}{2^4}+\text{etc.}$$
$$=\tfrac{1}{6}(\log 2)^3-\frac{\pi^2}{12}\log 2+\left(\frac{1}{1^3}+\frac{1}{3^3}+\frac{1}{5^3}+\text{etc.}\right).$$

(2) $$1+9\,.\,(\tfrac{1}{4})^4+17\,.\left(\frac{1\,.\,5}{4\,.\,8}\right)^4+25\,.\left(\frac{1\,.\,5\,.\,9}{4\,.\,8\,.\,12}\right)^4+\text{etc.}=\frac{2\sqrt{2}}{\sqrt{\pi}\,.\,\{\Gamma(\frac{3}{4})\}^2}.$$

(3) $$1-5\,.\,(\tfrac{1}{2})^3+9\,.\left(\frac{1\,.\,3}{2\,.\,4}\right)^3-\text{etc.}=\frac{2}{\pi}.$$

(4) $$\frac{1^{13}}{e^{2\pi}-1}+\frac{2^{13}}{e^{4\pi}-1}+\frac{3^{13}}{e^{6\pi}-1}+\text{etc.}=\frac{1}{24}.$$

(5) $$\frac{\coth\pi}{1^7}+\frac{\coth 2\pi}{2^7}+\frac{\coth 3\pi}{3^7}+\text{etc.}=\frac{19\pi^7}{56700}.$$

......

(11) $$\frac{2}{3}\int_0^1\frac{\tan^{-1}x}{x}dx-\int_0^{2-\sqrt{3}}\frac{\tan^{-1}x}{x}dx=\frac{\pi}{12}\log(2+\sqrt{3}).$$

......

* There is always more in one of Ramanujan's formulæ than meets the eye, as anyone who sets to work to verify those which look the easiest will soon discover. In some the interest lies very deep, in others comparatively near the surface; but there is not one which is not curious and entertaining.

"VI. Theorems on transformation of series and integrals, *e.g.*,

(1) $$\pi\left(\frac{1}{2}-\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}-\frac{1}{\sqrt{5}+\sqrt{7}}+\text{ etc.}\right)$$
$$=\frac{1}{1\sqrt{1}}-\frac{1}{3\sqrt{3}}+\frac{1}{5\sqrt{5}}-\text{ etc.}$$

......

(4) If $\int_0^a \phi(p, x)\cos nx\,dx = \psi(p, n)$, then
$$\frac{\pi}{2}\int_0^\infty \phi(p, x)\,\phi(q, nx)\,dx = \int_0^\infty \phi(q, x)\,\phi(p, nx)\,dx.$$

(5) If $\alpha\beta = \pi$, then $$\sqrt{\alpha}\int_0^\infty \frac{e^{-x^2}}{\cosh \alpha x}\,dx = \sqrt{\beta}\int_0^\infty \frac{e^{-x^2}}{\cosh \beta x}\,dx.$$

......

"VII. Theorems on approximate integration and summation of series.

(1) $1^2\log 1+2^2\log 2+3^2\log 3+\ldots+x^2\log x$
$$=\frac{x(x+1)(2x+1)}{6}\log x-\frac{x^3}{9}+\frac{1}{4\pi^2}\left(\frac{1}{1^3}+\frac{1}{3^3}+\ldots\right)+\frac{x}{12}-\frac{1}{360x}+\ldots\ldots$$

(2) $$1+\frac{x}{\underline{|1}}+\frac{x^2}{\underline{|2}}+\frac{x^3}{\underline{|3}}+\ldots+\frac{x^x}{\underline{|x}}\theta = \frac{e^x}{2}$$

where $\theta = \dfrac{1}{3}+\dfrac{4}{135(x+k)}$ where k lies between $\dfrac{8}{45}$ and $\dfrac{2}{21}$.

(3) $$1+\left(\frac{x}{\underline{|1}}\right)^5+\left(\frac{x^2}{\underline{|2}}\right)^5+\left(\frac{x^3}{\underline{|3}}\right)^5+\text{ etc.} = \frac{\sqrt{5}}{4\pi^2}\cdot\frac{e^{5x}}{5x^2-x+\theta},$$

where θ vanishes when $x=\infty$.

......

(5) $$\frac{1}{1001}+\frac{1}{1002^2}+\frac{3}{1003^3}+\frac{4^2}{1004^4}+\frac{5^3}{1005^5}+\text{ etc.}$$
$$=\frac{1}{1000}-10^{-440}\times 1{\cdot}0125 \text{ nearly.}$$

(6) $$\int_0^a e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}-\frac{e^{-a^2}}{2a\,+}\,\frac{1}{a\,+}\,\frac{2}{2a\,+}\,\frac{3}{a\,+}\,\frac{4}{2a\,+}\text{ etc.}$$

(7) The coefficient of x^n in $\dfrac{1}{1-2x+2x^4-2x^9+2x^{16}-\text{etc.}}$
$$=\text{ the nearest integer to } \frac{1}{4n}\left\{\cosh(\pi\sqrt{n})-\frac{\sinh(\pi\sqrt{n})}{\pi\sqrt{n}}\right\}.^*$$

* This is quite untrue. But the formula is extremely interesting for a variety of reasons.

"IX. Theorems on continued fractions, a few examples are:—

(1) $$\frac{4}{x+}\frac{1^2}{2x+}\frac{3^2}{2x+}\frac{5^2}{2x+}\frac{7^2}{2x+}\text{ etc.} = \left\{\frac{\Gamma\left(\frac{x+1}{4}\right)}{\Gamma\left(\frac{x+3}{4}\right)}\right\}^2.$$

......

(4) If $$u = \frac{x}{1+}\frac{x^5}{1+}\frac{x^{10}}{1}+\frac{x^{15}}{1}+\frac{x^{20}}{1}+\text{ etc.}$$

and $$v = \frac{\sqrt[5]{x}}{1}+\frac{x}{1+}\frac{x^2}{1}+\frac{x^3}{1}+\text{ etc.}$$

then $$v^5 = u\,.\,\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2+2u^3+u^4}.$$

(5) $$\frac{1}{1+}\frac{e^{-2\pi}}{1}+\frac{e^{-4\pi}}{1}+\frac{e^{-6\pi}}{1}+\text{ etc.} = \left(\sqrt{\frac{5+\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{2}\right)\sqrt[5]{e^{2\pi}}.$$

(6) $$\frac{1}{1-}\frac{e^{-\pi}}{1}+\frac{e^{-2\pi}}{1}-\frac{e^{-3\pi}}{1}+\text{ etc.} = \left(\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}-1}{2}\right)\sqrt[5]{e^{\pi}}.$$

(7) $\frac{1}{1+}\frac{e^{-\pi\sqrt{n}}}{1}+\frac{e^{-2\pi\sqrt{n}}}{1}+\frac{e^{-3\pi\sqrt{n}}}{1}+\text{ etc.}$ can be exactly found if n be any positive rational quantity. . . ."

February 27, 1913.

". . . I have found a friend in you who views my labours sympathetically. This is already some encouragement to me to proceed. . . . I find in many a place in your letter rigorous proofs are required, and you ask me to communicate the methods of proof. . . . I told him* that the sum of an infinite no of terms of the series $1+2+3+4+\ldots = -\frac{1}{12}$ under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal. . . What I tell you is this. Verify the results I give, and, if they agree with your results . . . you should at least grant that there may be some truths in my fundamental basis. . . .

"To preserve my brains I want food, and this is now my first consideration. Any sympathetic letter from you will be helpful to me here to get a scholarship, either from the University or from Government. . . .

1. The no of prime nos less than $e^a = \int_0^\infty \frac{a^x dx}{xS_{x+1}\Gamma(x+1)}$

where $$S_{x+1} = \frac{1}{1^{x+1}}+\frac{1}{2^{x+1}}+\ldots$$

* Referring to a previous correspondence.

2. The no of prime nos less than $n =$

$$\frac{2}{\pi}\left\{\frac{2}{B_2}\left(\frac{\log n}{2\pi}\right)+\frac{4}{3B_4}\left(\frac{\log n}{2\pi}\right)^3+\frac{6}{5B_6}\left(\frac{\log n}{2\pi}\right)^5+\text{ etc.}\right.$$

where $B_2 = \frac{1}{6}$; $B_4 = \frac{1}{30}$, etc., the Bernoullian nos. . . .

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The order of $\theta(x)$ which you asked in your letter is $\sqrt{\left(\frac{x}{\log x}\right)}$.

......

(1) If $F(x) = \frac{1}{1}+\frac{x}{1}+\frac{x^2}{1}+\frac{x^3}{1}+\frac{x^4}{1}+\frac{x^5}{1}+\text{ etc.}$

then $\left\{\frac{\sqrt{5}+1}{2}+e^{-2\alpha/5}F(e^{-2\alpha})\right\}\left\{\frac{\sqrt{5}+1}{2}+e^{-2\beta/5}F(e^{-2\beta})\right\}=\frac{5+\sqrt{5}}{2}$,

with the condition $\alpha\beta = \pi^2$. . . .

$$e.g.\ \frac{1}{1}+\frac{e^{-2\pi\sqrt{5}}}{1}+\frac{e^{-4\pi\sqrt{5}}}{1}+\text{ etc. }\ldots = e^{2\pi/\sqrt{5}}\left\{\frac{\sqrt{5}}{1+\sqrt[5]{5^{\frac{3}{4}}\left(\frac{\sqrt{5}-1}{2}\right)^{\frac{5}{2}}-1}}-\frac{\sqrt{5}+1}{2}\right\}$$

The above theorem is a particular case of a theorem on the c.f.

$$\frac{1}{1}+\frac{ax}{1}+\frac{ax^2}{1}+\frac{ax^3}{1}+\frac{ax^4}{1}+\frac{ax^5}{1}+\text{ etc.}$$

which is a particular case of the c.f.

$$\frac{1}{1}+\frac{ax}{1+bx}+\frac{ax^2}{1+bx^2}+\frac{ax^3}{1+bx^3}+\text{ etc.}$$

which is a particular case of a general theorem on c.f.

(2) i. $4\int_0^\infty \frac{xe^{-x\sqrt{5}}}{\cosh x}\,dx = \frac{1}{1}+\frac{1^2}{1}+\frac{1^2}{1}+\frac{2^2}{1}+\frac{2^2}{1}+\frac{3^2}{1}+\frac{3^2}{1}+\text{ etc.}$

ii. $4\int_0^\infty \frac{x^2e^{-x\sqrt{3}}}{\cosh x}\,dx = \frac{1}{1}+\frac{1^3}{1}+\frac{1^3}{3}+\frac{2^3}{1}+\frac{2^3}{5}+\frac{3^3}{1}+\frac{3^3}{7}+\text{ etc.}$

......

(5) $\frac{1^5}{e^{2\pi}-1}\,\frac{1}{2500+1^4}+\frac{2^5}{e^{4\pi}-1}\,\frac{1}{2500+2^4}+\ldots = \frac{123826979}{6306456}-\frac{25\pi}{4}\coth^2 5\pi.$

(6) If $v = \frac{x}{1}+\frac{x^3+x^6}{1}+\frac{x^6+x^{12}}{1}+\frac{x^9+x^{18}}{1}+\text{ etc.}$

then i. $x\left(1+\frac{1}{v}\right)=\frac{1+x+x^3+x^6+x^{10}+\text{ etc.}}{1+x^9+x^{27}+x^{54}+x^{90}+\text{ etc.}}$

ii. $x^3\left(1+\frac{1}{v^3}\right)=\left(\frac{1+x+x^3+x^6+{}^{10}+\text{ etc.}}{1+x^3+x^9+x^{18}+x^{30}+\text{ etc.}}\right)^4.$

......

(8) $$\int_0^\infty \frac{1+ab^2x^2}{1+x^2}\,\frac{1+ab^4x^2}{1+b^2x^2}\ldots\,dx = \frac{\pi}{2(1+b+b^3+b^6+\ldots)}\,\frac{1-ab^2}{1-ab}\,\frac{1-ab^4}{1-ab^3}\ldots$$

......

(12) If $\frac{1}{2}\pi\alpha = \log\tan(\frac{1}{4}\pi+\frac{1}{4}\pi\beta)$, then

$$\frac{1^2+\alpha^2}{1^2-\beta^2}\left(\frac{3^2-\beta^2}{3^2+\alpha^2}\right)^3\left(\frac{5^2+\alpha^2}{5^2-\beta^2}\right)^5\left(\frac{7^2-\beta^2}{7^2+\alpha^2}\right)^7\ldots = e^{\frac{1}{2}\pi\alpha\beta}.$$

......

(16) If $F(\alpha,\beta) = \tan^{-1}\left(\frac{\alpha}{x+}\,\frac{\beta^2+k^2}{x}\,\frac{}{+}\,\frac{\alpha^2+(2k)^2}{x}\,\frac{}{+}\,\frac{\beta^2+(3k)^2}{x}\ldots\right)$, then

$$F(\alpha,\beta)+F(\beta,\alpha) = 2F\left(\frac{\alpha+\beta}{2},\ \frac{\alpha+\beta}{2}\right).$$

(17) If $F(k) = 1+\left(\frac{1}{2}\right)^2 k + \left(\frac{1\,.\,3}{2\,.\,4}\right)^2 k^2+\ldots$ and $F(1-k) = \sqrt{(210)}\,F(k)$,

then $k = (\sqrt{2}-1)^4(2-\sqrt{3})^2(\sqrt{7}-\sqrt{6})^4(8-3\sqrt{7})^2(\sqrt{10}-3)^4(4-\sqrt{15})^4$
$\times(\sqrt{15}-\sqrt{14})^2(6-\sqrt{35})^2$.

(18) If $$F(\alpha) = 1+\frac{1\,.\,2}{3^2}\alpha+\frac{1\,.\,2\,.\,4\,.\,5}{3^2\,.\,6^2}\alpha^2+\ldots$$

and $$\frac{F(1-\alpha)}{F(\alpha)} = 5\frac{F(1-\beta)}{F(\beta)},$$

then $$\sqrt[3]{(\alpha\beta)}+\sqrt[3]{\{(1-\alpha)(1-\beta)\}}+3\sqrt[6]{\{\alpha\beta(1-\alpha)(1-\beta)\}} = 1.$$

......

April 17, 1913.

". . . I am a little pained to see what you have written. . . .* I am not in the least apprehensive of my method being utilised by others. On the contrary, my method has been in my possession for the last eight years, and I have not found anyone to appreciate the method. As I wrote in my last letter, I have found a sympathetic friend in you, and I am willing to place unreservedly in your hands what little I have. It was on account of the novelty of the method I have used that I am a little diffident even now to communicate my own way of arriving at the expressions I have already given. . . .

". . . I am glad to inform you that the local University has been pleased to grant me a scholarship of £60 per annum for two years, and this was at the instance of Dr. Walker, F.R.S., Head of the Meteorological Department in India, to whom my thanks are due. . . . I request you to convey my thanks also to Mr. Littlewood, Dr. Barnes, Mr. Berry, and others who take an interest in me. . . ."

* Ramanujan might very reasonably have been reluctant to give away his secrets to an English mathematician, and I had tried to reassure him on this point as well as I could.

III.

It is unnecessary to repeat the story of how Ramanujan was brought to England. There were serious difficulties; and the credit for overcoming them is due primarily to Prof. E. H. Neville, in whose company Ramanujan arrived in April, 1914. He had a scholarship from Madras of £250, of which £50 was allotted to the support of his family in India, and an exhibition of £60 from Trinity. For a man of his almost ludicrously simple tastes, this was an ample income; and he was able to save a good deal of money, which was badly wanted later. He had no duties, and could do as he pleased; he wished, indeed, to qualify for a Cambridge degree as a research student, but this was a formality. He was now, for the first time in his life, in a really comfortable position, and could devote himself to his researches without anxiety.

There was one great puzzle. What was to be done in the way of teaching him modern mathematics? The limitations of his knowledge were as startling as its profundity. Here was a man who could work out modular equations, and theorems of complex multiplication, to orders unheard of, whose mastery of continued fractions was, on the formal side at any rate, beyond that of any mathematician in the world, who had found for himself the functional equation of the Zeta-function, and the dominant terms of many of the most famous problems in the analytic theory of numbers; and he had never heard of a doubly periodic function or of Cauchy's theorem, and had, indeed, but the vaguest idea of what a function of a complex variable was. His ideas as to what constituted a mathematical proof were of the most shadowy description. All his results, new or old, right or wrong, had been arrived at by a process of mingled argument, intuition, and induction, of which he was entirely unable to give any coherent account.

It was impossible to ask such a man to submit to systematic instruction, to try to learn mathematics from the beginning once more. I was afraid, too, that, if I insisted unduly on matters which Ramanujan found irksome, I might destroy his confidence or break the spell of his inspiration. On the other hand, there were things of which it was impossible that he should remain in ignorance. Some of his results were wrong, and in particular those which concerned the distribution of primes, to which he attached the greatest importance. It was impossible to allow him to go through life supposing that all the zeros of the Zeta-function were real. So I had to try to teach him, and in a measure I succeeded, though obviously I learnt from him much more than he learnt from me. In a few years' time he had a very tolerable knowledge of the theory of functions and the analytic theory of numbers. He was never a mathematician of the modern school, and it was hardly desirable that he should become one; but he knew when he had proved a theorem and when he had not. And his flow of original ideas showed no symptom of abatement.

I should add here a word about Ramanujan's interests outside mathematics. Like his mathematics, they showed the strangest contrasts. He had very little interest, I should say, in literature as such, or in art, though he could tell good literature from bad. On the other hand, he was a keen philosopher, of what appeared, to followers of the modern Cambridge school, a rather nebulous kind, and an ardent politician, of a pacifist and ultra-radical type. He adhered, with a severity most unusual in Indians resident in England, to the religious observances of his caste; but his religion was a matter of observance and not of intellectual conviction, and I remember well his telling me (much to my surprise) that all religions seemed to him more or less equally true. Alike in literature, philosophy, and mathematics, he had a passion for what was unexpected, strange, and odd; he had quite a small library of books by circle-squarers and other cranks.

It was in the spring of 1917 that Ramanujan first appeared to be unwell. He went into the Nursing Home at Cambridge in the early summer, and was never out of bed for any length of time again. He was in sanatoria at Wells, at Matlock, and in London, and it was not until the autumn of 1918 that he showed any decided symptom of improvement. He had then resumed active work, stimulated perhaps by his election to the Royal Society, and some of his most beautiful theorems were discovered about this time. His election to a Trinity Fellowship was a further encouragement. Early in 1919 he had recovered, it seemed, sufficiently for the voyage home to India, and the best medical opinion held out hopes of a permanent restoration. I was rather alarmed by not hearing from him for a considerable time; but a letter reached me in February, 1920, from which it appeared that he was still active in research.

University of Madras,
January 12, 1920.

"I am extremely sorry for not writing you a single letter up to now. . . . I discovered very interesting functions recently which I call 'Mock' ϑ-functions. Unlike the 'False' ϑ-functions (studied partially by Prof. Rogers in his interesting paper), they enter into mathematics as beautifully as the ordinary ϑ-functions. I am sending you with this letter some examples. . . .

Mock ϑ-functions

$$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots$$

$$\psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots$$

Mock ϑ-functions (of 5th order)

$$f(q) = 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q)(1+q^3)} + \dots$$

Mock ϑ-functions (of 7th order)

$$\text{(i)}\quad 1+\frac{q}{1-q^2}+\frac{q^4}{(1-q^3)(1-q^4)}+\frac{q^9}{(1-q^4)(1-q^5)(1-q^6)}+\ldots$$

......"

He said little about his health, and what he said was not particularly discouraging; and I was quite unprepared for the news of his death.

IV.

Ramanujan published the following papers in Europe:—

(1) "Some Definite Integrals," 'Messenger of Mathematics,' vol. 44, pp. 10–18 (1914).

(2) "Some Definite Integrals connected with Gauss's sums," *ibid.*, pp. 75–85.

(3) "Modular Equations and Approximations to π," 'Quarterly Journal of Mathematics,' vol. 45, pp. 350–372 (1914).

(4) "New Expressions for Riemann's Functions $\zeta(s)$ and $\Xi(t)$," *ibid.*, vol. 46, pp. 253–261 (1915).

(5) "On Certain Infinite Series," 'Messenger of Mathematics,' vol. 45, pp. 11–15 (1915).

(6) "Summation of a Certain Series," *ibid.*, pp. 157–160.

(7) "Highly Composite Numbers," 'Proc. London Math. Soc.,' ser. 2, vol. 14, pp. 347–409 (1915).

(8) "Some Formulæ in the Analytic Theory of Numbers," 'Messenger of Mathematics,' vol. 45, pp. 81–84 (1916).

(9) "On Certain Arithmetical Functions," 'Trans. Cambridge Phil. Soc.,' vol. 22, No. 9, pp. 159–184 (1916).

(10) "Some Series for Euler's Constant," 'Messenger of Mathematics,' vol. 46, pp. 73–80 (1916).

(11) "On the Expression of Numbers in the Form $ax^2+by^2+cz^2+dt^2$," 'Proc. Cambridge Phil. Soc.,' vol. 19, pp. 11–21 (1917).

*(12) "Une formule asymptotique pour le nombre des partitions de n," 'Comptes Rendus,' January 2, 1917.

*(13) "Asymptotic Formulæ concerning the Distribution of Integers of various Types," 'Proc. London Math. Soc.,' ser. 2, vol. 16, pp. 112–132 (1917).

*(14) "The Normal Number of Prime Factors of a Number n," 'Quarterly Journal of Mathematics,' vol. 48, pp. 76–92 (1917).

*(15) "Asymptotic Formulæ in Combinatory Analysis," 'Proc. London Math. Soc.,' ser. 2, vol. 17, pp. 75–115 (1918).

*(16) "On the Coefficients in the Expansions of Certain Modular Functions," 'Roy. Soc. Proc.,' A, vol. 95, pp. 144–155 (1918).

(17) "On Certain Trigonometrical Sums and their Applications in the Theory of Numbers," 'Trans. Cambridge Phil. Soc.,' vol. 22, pp. 259–276 (1918).

(18) "Some Properties of $p(n)$, the Number of Partitions of n," 'Proc. Cambridge Phil. Soc.,' vol. 19, pp. 207–210 (1919).

(19) "Proof of Certain Identities in Combinatory Analysis," *ibid.*, pp. 214–216.

(20) "A Class of Definite Integrals," 'Quarterly Journal of Mathematics,' vol. 48, pp. 294–309 (1920).

(21) "Congruence Properties of Partitions," 'Math. Zeitschrift,' vol. 9, pp. 147–153 (1921).

Of these, those marked with an asterisk were written in collaboration with me, and (21) is a posthumous extract from a much larger unpublished

manuscript in my possession.† He also published a number of short notes in the 'Records of Proceedings at Meetings' of the London Mathematical Society, and in the 'Journal of the Indian Mathematical Society.' The complete list of these is as follows :—

Records of Proceedings at Meetings.

*(22) "Proof that almost all Numbers n are Composed of about $\log\log n$ prime factors," December 14, 1916.
*(23) "Asymptotic Formulæ in Combinatory Analysis," March 1, 1917.
(24) "Some Definite Integrals," Jan. 17, 1918.
(25) "Congruence Properties of Partitions," March 13, 1919.
(26) "Algebraic Relations between certain Infinite Products," March 13, 1919.

Journal of the Indian Mathematical Society.

(A) Articles and Notes.

(27) "Some Properties of Bernoulli's Numbers," vol. 3, pp. 219–235 (1911).
(28) "On Q. 330 of Prof. Sanjaná," vol. 4, pp. 59–61 (1912).
(29) "A Set of Equations," vol. 4, pp. 94–96 (1912).
(30) "Irregular Numbers," vol. 5, pp. 105–107 (1913).
(31) "Squaring the Circle," vol. 5, pp. 132–133 (1913).
(32) "On the integral $\int_0^x \arctan t \cdot \frac{dt}{t}$," vol. 7, pp. 93–96 (1915).
(33) "On the Divisors of a Number," vol. 7, pp. 131–134 (1915).
(34) "The Sum of the Square Roots of the First n Natural Numbers," vol. 7, pp. 173–175 (1915).
(35) "On the Product $\pi\left[1+\frac{x^2}{(a+nd)^2}\right]$," vol. 7, pp. 209–212 (1915).
(36) "Some Definite Integrals," vol. 11, pp. 81–88 (1919).
(37) "A Proof of Bertrand's Postulate," vol. 11, pp. 181–183 (1919).
(38) (Communicated by S. Narayana Aiyar), vol. 3, p. 60 (1911).

(B) Questions proposed and solved.

Nos. 260, 261, 283, 289, 294, 295, 298, 308, 353, 358, 386, 427, 441, 464, 489, 507, 541, 546, 571, 605, 606, 629, 642, 666, 682, 700, 723, 724, 739, 740, 753, 768, 769, 783, 785.

(C) Questions proposed but not solved as yet.

Nos. 284, 327, 359, 387, 441, 463, 469, 524, 525, 526, 584, 661, 662, 681, 699, 722, 738, 754, 770, 784, 1049, 1070, and 1076.

Finally, I may mention the following writings by other authors, concerned with Ramanujan's work :—

"Proof of a Formula of Mr. Ramanujan," by G. H. Hardy, 'Messenger of Mathematics,' vol. 44, pp. 18–21 (1915).

"Mr. S. Ramanujan's Mathematical Work in England," by G. H. Hardy (Report to the University of Madras, 1916. Privately printed).

"On Mr. Ramanujan's Empirical Expansions of Modular Functions," by L. J. Mordell, 'Proc. Camb. Phil. Soc.,' vol. 19, pp. 117–124 (1917).

"On Mr. Ramanujan's Congruence Properties of $p(n)$," by H. B. C. Darling, 'Proc. Cambridge Phil. Soc.,' vol. 19, pp. 217–218 (1919).

† All of Ramanujan's manuscripts passed through my hands, and I edited them very carefully for publication. The earlier ones I rewrote completely. I had no share of any kind in the results, except of course when I was actually a collaborator, or when explicit acknowledgment is made. Ramanujan was almost absurdly scrupulous in his desire to acknowledge the slightest help.

"Life Sketch of Ramanujan," editorial in the 'Journal of the Indian Math. Soc.,' vol. 11, p. 122 (1919).

"Note on the Parity of the Number which Enumerates the Partitions of a Number," by P. A. MacMahon, 'Proc. Cambridge Phil. Soc.,' vol. 20, pp. 281–283 (1921).

"Proof of certain Identities and Congruences Enunciated by S. Ramanujan," by H. B. C. Darling, 'Proc. London Math. Soc.,' ser. 2, vol. 19, pp. 350–372 (1921).

"On a Type of Modular Relation," by L. J. Rogers, *ibid.*, pp. 387–397.

It is plainly impossible for me, within the limits of a notice such as this, to attempt a reasoned estimate of Ramanujan's work. Some of it is very intimately connected with my own, and my verdict could not be impartial; there is much, too, that I am hardly competent to judge; and there is a mass of unpublished material, in part new and in part anticipated, in part proved and in part only conjectured, that still awaits analysis. But it may be useful if I state, shortly and dogmatically, what seems to me Ramanujan's finest, most independent, and most characteristic work.

His most remarkable papers appear to me to be (3), (7), (9), (17), (18), (19), and (21). The first of these is mainly Indian work, done before he came to England; and much of it had been anticipated. But there is much that is new, and, in particular, a very remarkable series of algebraic approximations to π. I may mention only the formulæ

$$\pi = \frac{63}{25}\,\frac{17+15\sqrt{5}}{7+15\sqrt{5}}, \qquad \frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2},$$

correct to 9 and 8 places of decimals respectively.

The long memoir (7) represents work, perhaps, in a backwater of mathematics, and is somewhat overloaded with detail; but the elementary analysis of "highly composite" numbers—numbers which have more divisors than any preceding number—is exceedingly remarkable, and shows very clearly Ramanujan's extraordinary mastery over the algebra of inequalities. Papers (9) and (17) should be read together, and in connection with Mr. Mordell's paper mentioned above; for Mr. Mordell afterwards proved a great deal that Ramanujan conjectured. They contain, in particular, very beautiful contributions to the theory of the representation of numbers by sums of squares. But I am inclined to think that it was in the theory of partitions, and the allied parts of the theories of elliptic functions and continued fractions, that Ramanujan shows at his very best. It is in papers (18), (19), and (21), and in the papers of Prof. Rogers and Mr. Darling, that I have quoted, that this side of his work (so far as it has been published) is to be found. It would be difficult to find more beautiful formulæ than the "Rogers-Ramanujan" identities, proved in (19); but here Ramanujan must take second place to Prof. Rogers; and, if I had to select one formula from all Ramanujan's work, I would agree with Major MacMahon in selecting a formula from (18), viz.,

$$p(4)+p(9)x+p(14)x^2+\ldots = 5\,\frac{\{(1-x^5)(1-x^{10})(1-x^{15})\ldots\}^5}{\{(1-x)(1-x^2)(1-x^3)\ldots\}^6},$$

where $p(n)$ is the number of partitions of n.

I have often been asked whether Ramanujan had any special secret; whether his methods differed in kind from those of other mathematicians; whether there was anything really abnormal in his mode of thought. I cannot answer these questions with any confidence or conviction; but I do not believe it. My belief is that all mathematicians think, at bottom, in the same kind of way, and that Ramanujan was no exception. He had, of course, an extraordinary memory. He could remember the idiosyncrasies of numbers in an almost uncanny way. It was Mr. Littlewood (I believe) who remarked that "every positive integer was one of his personal friends." I remember once going to see him when he was lying ill at Putney. I had ridden in taxicab No. 1729, and remarked that the number (7.13.19) seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways." I asked him, naturally, whether he knew the answer to the corresponding problem for fourth powers; and he replied, after a moment's thought, that he could see no obvious example, and thought that the first such number must be very large. His memory, and his powers of calculation, were very unusual, but they could not reasonably be called "abnormal." If he had to multiply two large numbers, he multiplied them in the ordinary way; he would do it with unusual rapidity and accuracy, but not more rapidly or more accurately than any mathematician who is naturally quick and has the habit of computation. There is a table of partitions at the end of our paper (15). This was, for the most part, calculated independently by Ramanujan and Major MacMahon; and Major MacMahon was, in general, slightly the quicker and more accurate of the two.

It was his insight into algebraical formulæ, transformations of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and I can compare him only with Euler or Jacobi. He worked, far more than the majority of modern mathematicians, by induction from numerical examples; all of his congruence properties of partitions, for example, were discovered in this way. But with his memory, his patience, and his power of calculation, he combined also a power of generalisation, a feeling for form, and a capacity for rapid modification of his hypotheses, that was often really startling, and made him, in his own peculiar field, without a rival in his day.

It is often said that it is much more difficult now for a mathematician to be original than it was in the great days when the foundations of modern analysis were laid; and no doubt in a measure it is true. Opinions may differ as to the importance of Ramanujan's work, the kind of standard by which it should be judged, and the influence which it is likely to have on the mathematics of the future. It has not the simplicity and the inevitableness of the very greatest work; it would be greater if it were less strange. One gift it has which no one can deny, profound and invincible originality. He would probably have been a greater mathematician if he had been

caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of greater importance. On the other hand, he would have been less of a Ramanujan, and more of a European professor, and the loss might have been greater than the gain.

G. H. H.

WOLDEMAR VOIGT, 1850—1919.

WOLDEMAR VOIGT was born at Leipzig in the year 1850. His school education was followed by attendance at the University there until his studies were interrupted by the outbreak of the Franco-German war of 1870, in which he served. His University course was afterwards completed at Königsberg in the years 1871–74. He here came under the influence of Franz Neumann, which determined largely the character of his own subsequent work. Alike in the type of subjects to which he devoted himself, in the formal elegance of his mathematical expositions, and in the severe precision of his style we can trace the inspiration of his illustrious teacher, for whom he retained a profound veneration. His abilities quickly gained recognition: he was made Extraordinary Professor at Königsberg in 1875, and was called to Göttingen as Ordinary Professor of Theoretical Physics in 1883. He held this post till his death on December 13, 1919.

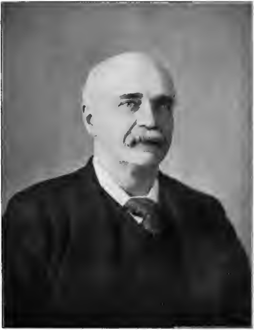
The complexion of his scientific work is, on the whole, indicated by the title of his chair. It was, however, always in close relation to phenomena, and he carried out a classical series of measurements on the elasticity of crystals which demanded the utmost precision. For experimental work, except on a minute scale, he appears to have had, till a late period, scanty facilities. His favourite province, which he cultivated with a life-long enthusiasm, was the physical properties of crystals. In particular his determinations of elastic constants of various crystals may claim to have set at rest a historical controversy. The special form of molecular hypothesis on which the theories of Poisson and Cauchy were based involved the conclusion that certain relations must exist between the constants of a crystal whatever its classification. This would reduce the number of constants in a general scheme to fifteen. Voigt succeeded in proving that the relations in question were, in many cases, not even approximately fulfilled. This may be regarded as a vindication of the attitude of Green and his followers in this country who, avoiding special hypothesis, postulated only the principle of energy. Voigt himself did not accept this point of view as final, and indicated lines on which the old molecular hypothesis might be amended. A masterly

review of the whole subject of crystalline elasticity was communicated by him to the Physical Congress held in Paris in 1900. This is distinguished by its orderly arrangement, and by the extreme elegance of the mathematical developments. Like Maxwell, he attached great importance to the classification and nomenclature of mathematical concepts; it is to him in particular that we owe the term "tensor," whose application has recently been so much extended. Voigt was also keenly interested in the other physical properties of crystals, and published an extensive treatise on 'Kristallphysik' (1910). He was the author of various papers on optical and electrical subjects, and of a systematic treatise on 'Magneto- und Elektro-optik' (1908). The breadth of his scientific sympathies may be further indicated by an allusion to his papers on Vortex Motion and on the Zeemann effect. In a paper on the Doppler effect published in 1887, Voigt was the first to establish the electromagnetic equations of transformation applicate to moving systems, which have played so important a part in the initial stages of the principle of relativity. A text-book in two volumes on Thermodynamics was dedicated to Lord Kelvin.

Voigt was a frequent visitor to this country, often as the delegate of the Göttingen Academy. He was present, for instance, at the Stokes jubilee in 1899, at the Owens College jubilee in 1902, at the Cambridge meeting of the British Association in 1904, at the St. Andrews 500th anniversary in 1911, and at the Royal Society celebration in 1912. No more acceptable representative could have been chosen. He had a generous appreciation of this country and its institutions, and an especial admiration of its leading physicists, in particular Stokes and Kelvin. He received many honorary degrees from British Universities, and was elected a Foreign Member of the Royal Society in 1913.

His many friends in this country felt an especial pang when the war came to interrupt intercourse with one whom they had found to be so sympathetic. It is to be recorded that even after the outbreak he made a courageous protest against the indiscriminate disparagement of everything English or French which was rife in Germany. He ventured to assert that there were elements in French and English culture which his own countrymen would do well at least to respect. On the general merits of the struggle he adopted the national view, which became embittered as the war went on. Those who knew him on his visits here will prefer to forget the alienation of the later years, and to recall only the grave and dignified courtesy, the friendly recognition, and the single-minded devotion to scientific truth which marked a noble and loveable personality.

H. L.



W. de W. Abney

L. Fletcher